

OLIMPADA DE MATEMATICA

ETAPA LOCAL

18 februarie 2012

BAREM

CLASA A VII-A

1.	Din oficiu	1p
	$\sqrt{5-x} \in \mathbb{N} \Rightarrow 5-x$ p trat perfect $\Rightarrow x \in \{5; 4; 1\}$	2 p
	$\sqrt{6-y} \in \mathbb{N} \Rightarrow 6-y$ p trat perfect $\Rightarrow y \in \{6; 5; 2\}$	2 p
	$\sqrt{13-z} \in \mathbb{N} \Rightarrow 13-z$ p trat perfect $\Rightarrow z \in \{13; 12; 9; 4\}$	2 p
	$\sqrt{x+y+z} \in \mathbb{N} \Rightarrow x+y+z$ p trat perfect	1 p
	$x=5; y=2; z=9$, sau $x=1; y=6; z=9$ sau $x=1; y=2; z=13$ care nu convine Rezult $\overline{xyz} = 529$ sau $\overline{xyz} = 169$	2 p
2.	Din oficiu	1p
a.)	$\frac{x}{0,(1)} = \frac{y}{0,(3)} \Rightarrow \frac{x}{1} = \frac{y}{3}$	1p
	$\frac{y}{0,(5)} = \frac{z}{0,(7)} \Rightarrow \frac{y}{5} = \frac{z}{7} \Rightarrow$	1p
	$\frac{x}{5} = \frac{y}{15} = \frac{z}{21} = \frac{x+y+z}{41} = 3 \Rightarrow x=15, y=45, z=63$	3p
b.)	Cum $5 \cdot 9 = 45$ i $(5,9) = 1$ rezult c $A:45$ dac $A:5$ i $A:9$	1p
	$A = 5^n \cdot 5^3 \cdot 2^n - 125 = 125 \cdot (5^n \cdot 2^n - 1) = 125 \cdot (10^n - 1) = 125 \cdot \underbrace{99\dots 9}_{n-1}$	2p
	Cum $A = 125 \cdot \underbrace{99\dots 9}_{n-1} : 5$ i $A = 125 \cdot \underbrace{99\dots 9}_{n-1} : 9 \Rightarrow A:45$	1p
3.	Din oficiu	1p
	Ipotez , concluzie i desen	1 p
	$\hat{\text{In}} \triangle ABC$ (BE bis. $\Rightarrow \frac{CE}{15} = \frac{AE}{20} = \frac{CE+AE}{15+20} = \frac{AC}{35}$)	3 p
	$\frac{CE}{AC} = \frac{15}{35} = \frac{3}{7}$ (1)	2 p
	$\hat{\text{In}} \triangle ADC$ $EF \parallel AD \Rightarrow \triangle CFE \sim \triangle CDA$	1 p
	$\frac{EF}{AD} = \frac{CE}{AC}$ (2)	1 p
	Din rela iile (1) i (2) rezult $\frac{EF}{14} = \frac{3}{7} \Rightarrow EF = 6cm$	1 p

4.	Din oficiu	1p
	Ipotez , concluzie i desen	1 p
	$\hat{\text{În}} \triangle BEC, AF \parallel EC \text{ (cf. } T.Th.) \Rightarrow \frac{BF}{FC} = \frac{BA}{AE} \Leftrightarrow \frac{BC}{FC} = \frac{BE}{AE}$	3p
	$\hat{\text{În}} \triangle DBC, AF \parallel DB \text{ (cf. } T.Th.) \Rightarrow \frac{FC}{FB} = \frac{CA}{AD} \Leftrightarrow \frac{BC}{FB} = \frac{DC}{AD}$	3p
	$\frac{DA}{DC} \cdot \frac{FC}{FB} \cdot \frac{EB}{EA} = \frac{FB}{BC} \cdot \frac{FC}{FB} \cdot \frac{BC}{FC} = 1$	2p